An experimental investigation of density-wave-type oscillations in a convective boiling upflow system

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Two-phase flow density-wave-type oscillations in a single-channel, high-pressure forced-convection boiling upflow system were investigated experimentally using water as working fluid. Heat flux and quality onset of density-wave-type oscillations, defined in the text as limiting heat flux and limiting quality, were determined within the ranges of the experiments. Their dependence on system pressure, mass flux, inlet subcooling, and the size of exit restriction was found and presented graphically. Based on these findings, a correlation is formulated for the prediction of limiting heat flux and quality of density-wave-type oscillations.

Keywords: instabilities; two-phase flow instabilities; density-wave-type oscillations

Introduction

Flow instability induced by boiling heat transfer is a very important phenomenon in two-phase flow systems, such as nuclear steam generators, boilers, refrigeration plants, and various two-phase-flow heat exchangers. Two-phase flow instabilities can cause heat transfer crisis, structure vibration, and system control problems. Therefore, flow instabilities must be evaluated fully to ensure system operation and safety. Stenning (1964) investigated and identified three types of dynamic instabilities, namely, density-wave type, pressure-drop type, and thermal oscillations in a single-channel upflow boiling system. However, the most common type of dynamic two-phase flow instability encountered in industry is the density-wave-type instability. Mayinger (1968) also identified thermal oscillations in the 1960s. Dynamic instabilities were also identified by Kakaç et al. (1977). Pakopoulos et al. (1980) carried out a study of the dynamics of two-phase flow in vapor generators, focusing on the effect of coupling between heat-flux and mass flow rate on density-wave-type instability. A distributed-parameter, transient theoretical analysis of the conservation equations of two-phase flow was presented. Mentes et al. (1989) carried out an experimental study on the effects of heat transfer augmentation and inlet subcooling on two-phase-flow dynamic instabilities. They found that the amplitudes and periods of the thermal oscillations increased with an increase in inlet subcooling. Kakaç et al. (1990) presented investigations of thermal instabilities in forced-convection boiling in a single vertical channel. Experiments with various heat inputs, inlet temperatures, and different-size heater tubes were conducted. A theoretical model was also developed, and the model's predictions of the periods and amplitudes were in good agreement with experimental results. Various reviews of two-phase flow instabilities with recent research on dynamic instabilities are cited in the reference section of this paper (see Bergles 1977; Bouré et al. 1973; Kakaç and Liu 1991). Most experimental research has been carried out on low-pressure systems with chlorofluorocarbon (CFC) refrigerants as the working fluid, for which the oscillation conditions are quite different from those encountered in an industrial system. Although some recent experimental studies by Xiao et al. (1993) have been carried out on high-pressure systems with water as the working fluid, more experimental results are still needed on dynamic instabilities for industrial equipment design and operation.

In this paper, the experimental results of density-wave-type instabilities in a high-pressure, forced-flow boiling system are presented. A series of experiments has been carried out in a high-pressure loop with water as the working fluid. The effects of system parameters on density-wave-type oscillation have been studied over a wide range. The experimental results have been correlated empirically to obtain the threshold of the density-wave-type oscillation. The results shown in this paper indicate that the density-wave-type oscillation is the most commonly encountered oscillation, with periods of about 1.5 to 4.5 seconds. Density-wave-type oscillations would induce thermal oscillations, which are quite critical in the design of two-phase flow systems.

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Experimental apparatus and method

Description of experimental setup

This work was carried out on a full-size, high-pressure water-vapor in-tube boiling system, as shown in Figure 1. The setup was built and used in order to investigate the basic mechanisms of different types of oscillations and the effects of various parameters on these oscillations.

Demineralized and degasified water was fed to the test section (7) via a surge tank (6) after being pressurized by a

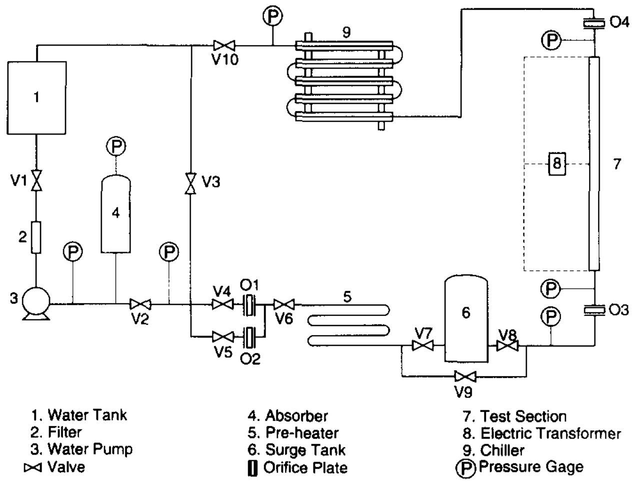

Figure 1 High-pressure water-vapor experiment setup

plunger pump (3). A bypass controlled by valve V3 was also installed in the system to ensure proper flow rate to the test section.

Water flow rate to the test section was controlled by the regulation of valves V4 and V5. Water was heated to the desired inlet temperature in the pre-heater (5), and then heated again in the test section until boiling occurred. The two-phase mixture was sent back to the water tank (1) through a chiller (9), where the mixture was condensed back to below 50°C.

To eliminate the flow oscillation generated by the plunger pump, an absorber (4) was added to the system right after the pump.

The diameter of the surge tank was 175 mm with a wall thickness of 13 mm and a height of 1500 mm. The surge tank was charged with nitrogen, which formed a compressible volume for the test section. This tank was an important dynamic component of the loop, providing the necessary compressible volume for the pressure-drop-type oscillations to occur. The working fluid (water) could also bypass the surge tank and enter the test section directly via valve V9 when valves V7 and V8 were closed.

The test section was made of stainless steel tubing (∅16 mm × 2 mm) with a length of 7.8 m. The heated length, however, was only 3.8 m. Heating was realized by applying AC power directly to the tube, which was insulated with fiberglass material. The heater was electrically insulated from the rest of the system. The test section terminated at an exit restriction.

An orifice plate (04) was installed after the test section to form the exit restriction. Three plates with different orifice diameters were used in turn to study the effect of the exit restriction on boiling instabilities.

Experimental measurements and uncertainties

Instruments were provided to measure the flow rate, heat input, pressures, and temperatures at various locations.

Temperature. For temperature measurements, standard chromel-constantan thermocouples were used. Twenty-eight pairs of these thermocouples were spot welded evenly along the heater tube wall, with 14 on one side and 14 directly across on the other. Fluid temperatures were measured with the same type of thermocouples by placing them in the midstream of the fluid channel. A typical uncertainty associated with temperature measurement was 0.375 percent.

Flow rate. Measurement of the flow rate was made at several locations in the system, marked by 01, 02, 03, and 04 in Figure 1. At each location, an orifice plate and a differential pressure transducer were used. This type of transducer had a response time of the order 0.1 seconds, and was considered accurate enough for dynamic measurements. A typical density-wave oscillation period encountered in the study was 1.5 seconds. The uncertainty of flow rate measurement was 2.64 percent.

Pressure. Pressures were measured by Bourdon-type pressure gauges at seven different locations. In addition to these gauges, a strain-gauge-type pressure transducer was used to measure and record the instantaneous pressure at the heater inlet.

Pressure measurements were made along the line at locations marked by P in Figure 1. The uncertainty associated with this type of pressure transducer was 5.1 percent.

Experimental procedure

Steady-state characteristics of the system were first obtained and plotted on the pressure drop versus mass-flux plane. These characteristic curves were used to mark the instability boundaries.

To study the effects of system pressure, mass flux, heat flux, inlet subcooling, and the size of exit restriction on density-wave-type oscillations, the following ranges were used to cover a broad two-phase flow region:

- System pressure: 30–100 bar
- Mass flux: 600–1300 kg/m^2.s

Notation

A	Constant
B	Constant
D	Inner diameter of the heater tube, m
d_i	Diameter of the exit restriction, m
G	Mass flux, kg/m^2 s
h	Enthalpy of working fluid, kJ/kg
h_1	Enthalpy of saturated liquid at exit pressure, kJ/kg
h_o	Enthalpy at inlet, kJ/kg
h_s	Saturation enthalpy, kJ/kg
Δh_v	Latent heat of evaporation, kJ/kg
L_h	Heated length of the tube, m
N_p	Dimensionless number
N_{TS}	Dimensionless number
N_Q	Dimensionless number
P_e	Exit pressure, bar
p	System pressure, bar
q	Heat input, kW
Q_0	Heat flux, kW/m^2
Q_c	Limiting heat flux, kW/m^2
T	Wall temperature, °C
x	Vapor quality
x_c	Limiting vapor quality

Greek symbols

β	Exit restriction diameter ratio, d_i/D
ρ_l	Liquid density, kg/m^3

- Inlet subcooling: 10–90°C
- Exit restriction diameter ratio, β: 0.33, 0.417, 0.5
- Heat flux: 0–700 kW/m^2

The experiment was conducted by fixing the mass flux at a desired value with a pre-selected system pressure and exit restriction, while increasing the heat flux to the test section. Then the preheater was turned on so that a desired inlet subcooling condition could be reached at the inlet of the test section. Next, the heat flux to the test section was increased to a starting value. Various data (pressure, temperature, flow variation) were taken after thermal equilibrium was attained in the system. Thereafter, heat flux was increased by a small increment repeatedly until the dry-out point was reached, or until the heater wall temperature became too high.

The above procedure was repeated by changing other parameters alternatively.

As the heat flux was increased during the process, the working fluid (water) in the test section approached its saturation temperature. At different times, sustained pressure-drop-type, density-wave-type, and thermal oscillations appeared. Whether the system was stable or not was judged by observing the real-time measurement of the data of interest. Every time the heat flux was increased, the system was disturbed, and the pressure and flow rate would start to oscillate. If this type of oscillation vanished, the system was said to be stable; if the oscillations were sustained, then the system was deemed to be unstable.

To determine the boundaries between the stable and unstable regions and between different oscillation modes, the increase in heat flux was adjusted by a very small amount. In the vicinity of the expected boundary or after reaching the unstable region, the heat flux was first decreased into the stable region and then increased very slowly to locate the instability boundary.

We defined the minimum heat-flux onset of density-wave-type oscillations as the limiting heat flux, Q_c, and the quality at the same time as the limiting quality, x_c. The term *threshold* or *threshold values* is also used to replace the limiting heat flux and quality where applicable.

Results and discussion

Density-wave-type oscillations have been observed to occur at low mass flow rates in the positive slope region along the steady-state characteristic curves. The system pressure drop was defined as the pressure drop from the surge tank to the exit of the exit restriction.

At low vapor qualities, the system was stable. As the heat input was increased and the quality was raised to a certain value in the test section, oscillations of mass flux, inlet pressure, and wall temperature with periods of 1.5 to 4.5 seconds were observed. These are density-wave-type oscillations. At first, the period of oscillation was about 4.5 seconds, and the oscillation amplitude of mass flux was quite large. The oscillation period and amplitude decrease with increase in heat input. During density-wave-type oscillations, the mass flux oscillation was always accompanied by a wall-temperature oscillation. The amplitudes of the wall-temperature oscillations were in the range of 10 to 30°C, and tended to increase as heat input was increased. A typical density-wave-type oscillation is shown in Figure 2. Density-wave-type oscillations generally have higher frequencies and smaller amplitudes as compared to other types of oscillations (Kakaç and Liu 1991).

The density-wave-type oscillations are related to kinematic wave propagation phenomena. In two-phase flow systems, temperature or enthalpy perturbations cause density or void fraction perturbations, which travel at the kinematic-wave velocity of the mixture. Therefore, fluid waves of alternately higher and lower densities travel across the system. It takes one high and one low wave to make a cycle, so it can be concluded that the periods of the density-wave-type oscillations are approximately equal to twice the residence time of a fluid particle passing through the system.

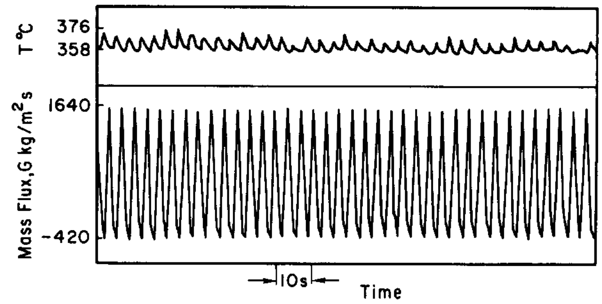

Figure 2 A typical density-wave-type oscillation ($P = 100$ bar, $G = 937.5$ kg/m^2.s, $T_{sub} = 90$°C, $\beta = 0.33$, $Q = 580.5$ kW/m^2)

The effect of parameters

The threshold values, i.e., limiting heat flux and quality, of the density-wave-type oscillations were found to be affected by the following parameters in different ways.

System pressure. This study showed that at higher system pressures, the values of limiting heat flux and quality were increased, as shown in Figure 3. This fact actually indicates that the system becomes more stable, since the system can sustain higher heat input without experiencing density-wave-type oscillations. Higher system pressure effectively reduces the density difference between the vapor and liquid phases.

Inlet mass flow rate. The threshold of the density-wave-type oscillations was also obtained as a function of mass flow rate under different system pressures, as shown in Figure 4. The limiting heat flux is increased as mass flux is increased. This leads the two-phase flow system to become more stable. The limiting quality, however, is reduced at the same time.

Inlet subcooling. The effect of the inlet subcooling on the threshold values is quite complex. Under lower system pressure (30 bars), this effect is monotonous. Both limiting heat flux and quality increase almost linearly with increase in subcooling, as shown in Figure 5. Under higher system pressures (50 bars and

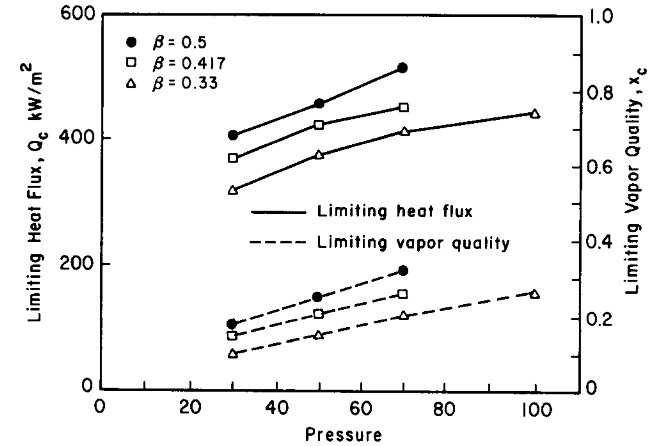

Figure 3 The effect of system pressure ($G = 937.5$ kg/m^2.s, $T_{sub} = 60$°C)

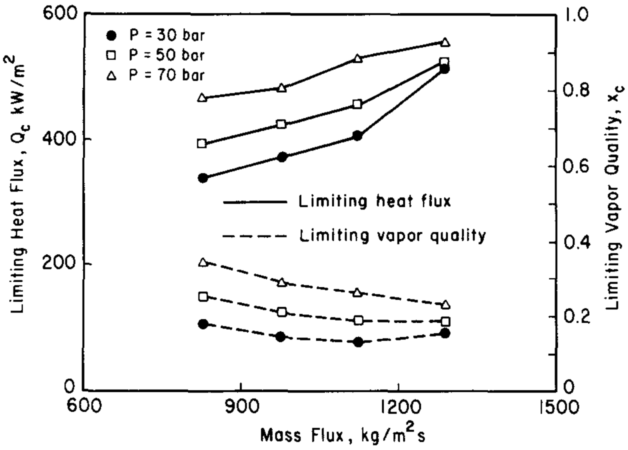

Figure 4 The effect of the inlet mass flow rate ($T_{sub} = 60°C$, $\beta = 0.417$)

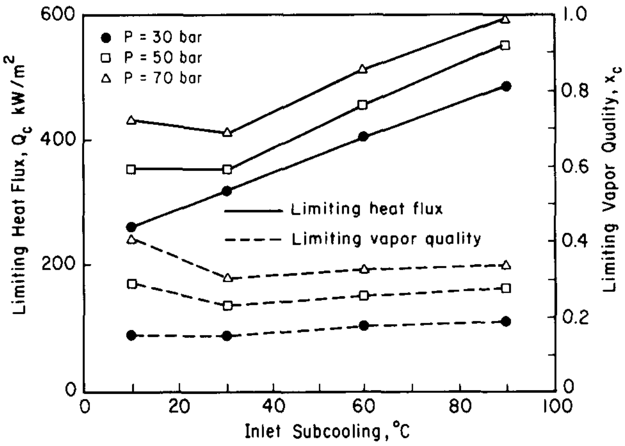

Figure 5 The effect of the inlet subcooling ($G = 937.5$ kg/m^2.s, $\beta = 0.5$)

above), however, the relationship becomes multivalued. Minima on both curves are found to occur at a subcooling of 30°C. This point is shifted to 60°C when using the exit restriction of $\beta = 0.33$. With increased inlet subcooling, the single liquid-phase region will expand downstream, which makes the system more stable. Meanwhile, the average vapor quality in the heater will decrease, which in turn reduces the response time for disturbances; therefore, the mass flux can undergo oscillations more easily (Wang 1990).

Exit restriction. The presence of the exit restriction increases the pressure drop in the two-phase flow region, as well as the change of pressure difference induced by phase change. When the orifice diameter decreases, the system becomes less stable. To make the system stable, an orifice with a larger diameter must be used (see Figure 3). The period of the density-wave-type oscillations is controlled by the residence time of the mixture within the system. The oscillation periods decrease with the increase of heat flux and orifice diameter, and increase with the increase of inlet subcooling and system pressure.

Prediction of the threshold of density-wave-type oscillations

It is clear from the experimental studies that the effects of various system parameters on limiting heat flux and quality are quite complicated. While further and extensive studies are needed, a relatively simple correlation to predict the threshold of this type of oscillation can be obtained from the present experimental study.

If one nondimensionalizes the energy equation for homogeneous two-phase flow, a dimensionless number N_Q can be obtained (see appendix):

$$N_Q = \frac{4Q_0 L_h}{h_l GD} \tag{1}$$

Replacing Q_o, which is the heat flux of stable flow, by the limiting heat-flux value, Q_c, and replacing h_l by $(h_s - h_0)$, we obtain

$$N_Q = \frac{4Q_c L_h}{(h_s - h_0)GD} \tag{2}$$

N_Q can therefore be considered as a ratio relating the total heated length L_h to the length occupied by the single liquid phase in the test section. It also shows the relationship between limiting heat flux and fluid inlet subcooling.

Multiplying both sides of Equation 2 by the limiting quality x_c, the following criterion to predict the threshold of the density-wave-type oscillation can be obtained:

$$x_c N_Q = x_c \frac{4Q_c L_h}{(h_s - h_0)GD} \tag{3}$$

Another dimensionless number N_P was derived from the dimensionless form of the momentum equation for homogeneous two-phase flow (see appendix):

$$N_P = \frac{P_e \rho_l}{G^2 L_h} \tag{4}$$

This number was used to show the effects of pressure and mass flux on the density-wave-type oscillations.

In addition, thermodynamic properties of the fluid were also taken into account:

$$N_{TS} = \frac{h_s - h_0}{\Delta h_v} \tag{5}$$

Equation 5 is used to consider the effect of inlet subcooling.

A simple correlation was thus obtained by introducing the following relationship among Equations 3, 4, and 5:

$$x_c N_Q = A + B\frac{N_P}{N_{TS}} \tag{6}$$

In practice, quality, x, in the system is to be found from the thermal equilibrium condition, and N_Q is to be calculated from Equation 1. The product of x and N_Q is then compared with the value calculated from Equation 6. If $xN_Q > x_c N_Q = A + BN_P/N_{TS}$, the system is then in the region where density-wave-type oscillations can occur. If $x_c N_Q < A + BN_P/N_{TS}$, the system will not experience this type of oscillation.

The constants A and B in Equation 6 are found to be functions of the exit restriction:

$\beta = 0.33$, $A = 0.2492$, $B = 0.05902$

$\beta = 0.42$, $A = 0.2404$, $B = 0.12020$

$\beta = 0.50$, $A = 0.2015$, $B = 0.20370$

Experimental data and those calculated from the above correlation are plotted in Figure 6, which shows a clear linear relationship between them. This linear relationship suggests that the correlation of Equation 6 can be used satisfactorily to predict the threshold values of heat flux and quality of the

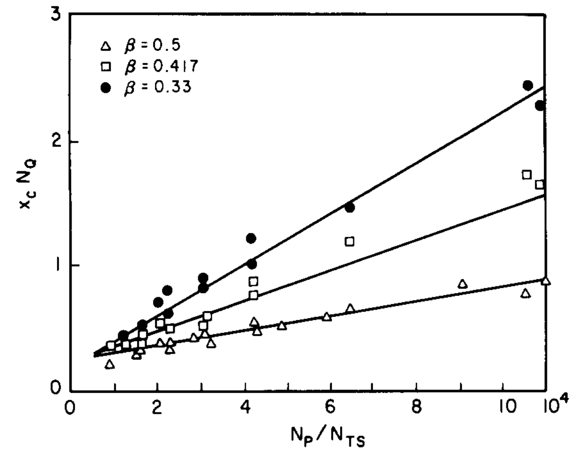

Figure 6 Comparison of correlation with experimental data

density-wave-type oscillations in a high-pressure upflow boiling system under the present experimental conditions.

Concluding remarks

(1) In a boiling two-phase flow system, the occurrence of the density-wave-type oscillations is affected by system pressure, inlet subcooling, mass flux, and the size of exit restriction. These effects have been given a full discussion.
(2) The period of the density-wave-type oscillation is related to the residence time of the mixture particles traveling along the system. The amplitude of mass flow-rate oscillation is quite large, and flow reversal also occurs during the oscillations.
(3) During the density-wave-type oscillations, wall temperature also experiences oscillations. Under the present experimental conditions, the amplitude of temperature oscillations varied from 10°C to 30°C.

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Appendix

The dimensionless number, N_p, is introduced to consider the effects of system pressure and mass flux on density-wave-type oscillations in a two-phase-flow boiling system. This number can be obtained from the homogeneous-flow momentum equation:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial z} = -\frac{\partial P}{\partial z} - g\rho \tag{A.1}$$

We define the dimensionless quantities

$$\bar{\rho} = \frac{\rho}{\rho_l},\ \bar{u} = \frac{u}{u_0},\ \bar{t} = \frac{t}{L_h/u_0},\ \bar{Z} = \frac{z}{L_h},\ \bar{P} = \frac{P}{P_e},\ \bar{g} = \frac{g}{U^2/L_h}$$

and substituted into Equation A.1 to get

$$\frac{\partial(\bar{\rho}\bar{u})}{\partial \bar{t}} + \frac{\partial(\bar{\rho}\bar{u}^2)}{\partial \bar{Z}} = -\frac{P_e}{\rho_l u_0^2 L_h}\frac{\partial \bar{P}}{\partial \bar{Z}} - \bar{g}. \tag{A.2}$$

N_p is defined as

$$N_P = \frac{P_e}{\rho_l u_0^2 L_h} = \frac{P_e}{G^2 L_h} \tag{A.3}$$

The dimensionless number N_{TS} is used to consider the effect of inlet subcooling on the density-wave-type oscillation, and is defined as

$$N_{TS} = \frac{h_s - h_0}{\Delta h_V} \tag{A.4}$$

The threshold values of density-wave-type oscillations are determined in terms of the limiting values of both heat flux and vapor quality.

The homogeneous energy equation for two-phase flow can be written as

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial(\rho u h)}{\partial z} = Q''' \tag{A.5}$$

If we define the dimensionless quantities

$$\bar{\rho} = \frac{\rho}{\rho_l},\ \bar{u} = \frac{u}{u_0},\ \bar{t} = \frac{t}{L_h/u_0},\ \bar{h} = \frac{h}{h_l},\ \bar{Q} = \frac{Q'''}{Q_0'''}$$

and substitute them into Equation A.5, we obtain

$$\frac{\partial(\bar{\rho}\bar{h})}{\partial \bar{t}} + \frac{\partial(\bar{\rho}\bar{u}\bar{h})}{\partial \bar{Z}} = \frac{Q_0''' L_h}{\rho_l u_0 h_l}\bar{Q} \tag{A.6}$$

The dimensionless number N_Q is defined as the coefficient of the right-hand-side term in Equation A.6:

$$N_Q = \frac{Q_0''' L_h}{\rho_l u_0 h_l} \tag{A.7}$$

where

$$Q''' = \frac{4q}{D^2 L}$$

and

$$Q_0 = \frac{q}{DL}$$

Therefore, N_Q can be rewritten as

$$N_Q = \frac{4Q_0 L_h}{GDh_l} \tag{A.8}$$

Replace Q_0 with the limiting heat flux Q_c and h_l with $h_s - h_0$ to obtain

$$N_Q = \frac{4Q_c L_h}{G(h_s - h_0)D} \tag{A.9}$$